Listing's and Donders' Laws and the Estimation of the Point-of-Gaze

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Abstract

This paper examines the use of Listing's and Donders' laws for the calculation of the torsion of the eye in the estimation of the point-of-gaze. After describing Listing's and Donders' laws and providing their analytical representation, experimental results obtained while subjects looked at a computer screen are presented. The experimental results show that when the point-of-gaze was estimated using Listing's and Donders' laws there was no significant accuracy improvement relative to when eye torsion was ignored. While for a larger range of eye rotation the torsion would be more significant and should be taken into account, the torsion predicted by Listing's and Donders' laws may be inaccurate, even in ideal conditions. Moreover, eye torsion resulting from lateral head tilt can be significantly larger than the torsion predicted by Listing's and Donders' laws, and even have opposite direction. To properly account for eye torsion, it should be measured independently (e.g., by tracking the iris pattern and/or the scleral blood vessels).

Keywords: kinematics of the eye, eye torsion, Listing's law, Donders' law, gaze estimation accuracy

1 Introduction

The point-of-gaze can be defined as the intersection of the visual axis with the scene. The visual axis is the line defined by the center of the fovea and the nodal point of the eye, and it exhibits a subject-specific deviation from the optic axis of the eye of as much as 5º. Gaze estimation methods based on 3-D models (e.g., [Shih and Liu 2004; Guestrin and Eizenman 2006; Villanueva and Cabeza 2007; Guestrin and Eizenman 2008; Nagamatsu et al. 2008; Guestrin 2010]) follow three steps: (i) the reconstruction of the optic axis of the eye in 3-D space from a set of eye features (e.g., the pupil and corneal reflections); (ii) the reconstruction of the visual axis in 3-D space from the optic axis of the eye and the 3-D angular deviation between the visual and optic axes (obtained through a personal calibration procedure); (iii) the estimation of the point-of-gaze as the intersection of the visual axis with the 3-D scene.

Different gaze directions are associated with different amounts of torsion of the eye about the line connecting the center of rotation of the eye and the point-of-gaze (fixation line, line of fixation or fixation axis). Consequently, the torsion of the eye should be taken into account to accurately reconstruct the visual axis in 3-D space from the optic axis of the eye and the 3-D angle between the visual and optic axes that was obtained through personal calibration. Recent publications (e.g., [Villanueva and Cabeza 2007; Nagamatsu et al. 2008]) suggested using Listing's and Donders' laws to estimate eye torsion with the objective of improving the point-of-gaze estimation accuracy. The goals of this paper are (a) to quantify the difference in the point-of-gaze estimation accuracy when eye torsion is estimated with Listing's and Donders' laws relative to when eye torsion is ignored, and (b) to discuss the limitation of Listing's and Donders' laws in general gaze estimation applications.

2 Listing's and Donders' Laws

Under a certain set of conditions (the head is erect and stationary, the point-of-gaze is stationary, monocular viewing conditions with the gaze at infinity), the torsion of the eye follows approximately Listing's and Donders' laws [Carpenter 1977; Alpern 1969b; Ferman et al. 1987a]. Before stating these laws, it is necessary to define the primary position of the eye. The primary position can be defined as the position of the eyes in the skull when the head is in a natural erect position and the fixation line is horizontal and perpendicular to the line connecting the centers of rotation of both eyes. Note that, for most practical situations, the fixation line is very close to the visual axis.

Listing's law states that each movement of the eyes from the primary position to any other position is described by a single rotation about an axis that is perpendicular to the plane that contains the initial and final positions of the fixation line. Since all possible axes of rotation are perpendicular to the fixation line in the primary position, they are all contained in a vertical plane called the equatorial plane or Listing's plane. In this context, any position of the eye can be fully described by specifying the orientation of the axis of rotation in Listing's plane and the magnitude of the rotation from the primary position. Therefore, two degrees of freedom suffice to describe any eye position completely.

A consequence of Listing's law is that, except for purely horizontal or purely vertical movements, the meridian of the eye that is vertical in the primary position is systematically tilted with respect to the objective vertical (gravity) in any other position (tertiary position). Based on this, the primary position can be defined as the position from which purely horizontal movements or purely vertical movements are not associated with any tilt of the vertical meridian of the eye with respect to the objective vertical [Carpenter 1977; Alpern 1969a].

Donders' law states that the angle of the tilt of the vertical meridian of the eye with respect to the objective vertical at any given eye position is always the same regardless of the way the eyes reach that position. This implies that the torsion of the eye in any position is the same as would be observed if the eye moved to it from the primary position by rotation about a single axis in Listing's plane.

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To quantify the difference in the point-of-gaze estimation accuracy when eye torsion is estimated using Listing's and Donders' laws relative to when eye torsion is ignored, Listing's law has to be expressed analytically.

3 Analytical Expressions

In order to derive the analytical expressions, assume that the fixation line is parallel to the visual axis (or, equivalently, redefine the fixation line as the line that goes through the center of rotation of the eye and is parallel to the visual axis). This is strictly true when the gaze is at infinity, in which case the fixation line and the visual axis are about 0.4-0.8 mm apart. When the point-of-gaze is at a finite but relatively large distance, the assumption that the fixation line and visual axis are parallel is reasonable (e.g., for a point-of-gaze at 57 cm from the eye, the directions of the fixation line and the visual axis differ by less than 5 minutes of arc).

Let $\mathbf{c}$ be the nodal point of the eye/center of curvature of the cornea, $\boldsymbol{\omega}$ be the unit vector in the direction of the optic axis of the eye, $\mathbf{v}$ be the unit vector in the direction of the visual axis, and the subscript "p.p." represent the primary position of the eye (all coordinates are with respect to a world coordinate system).

The visual axis in 3-D space is then given by

$$\mathbf{g} = \mathbf{c} + k_g \mathbf{v} \ , \ k_g \in \Re , \tag{1}$$

and the point-of-gaze is given by the value of k_g corresponding to the intersection of the visual axis with the scene.

The rotation from the primary position ($\mathbf{v}_{\text{p.p.}}$) to any other position ($\mathbf{v}$) as prescribed by Listing's law is described by the rotation vector $\boldsymbol{\Gamma} = \varepsilon\, \boldsymbol{\lambda}$, where

$$\boldsymbol{\lambda} = \frac{\mathbf{v}_{\text{p.p.}} \times \mathbf{v}}{\left\| \mathbf{v}_{\text{p.p.}} \times \mathbf{v} \right\|} \tag{2}$$

is the unit vector in the direction of the axis of rotation according to the right-hand rule and

$$\varepsilon = \arccos(\mathbf{v}_{\text{p.p.}} \bullet \mathbf{v}) \tag{3}$$

is the magnitude of the rotation relative to the primary position (eccentricity angle). The corresponding rotation matrix, $\mathbf{R}_{\text{Listing}}$, can be readily obtained using Rodrigues' rotation formula [Belongie]. Then,

$$\mathbf{v} = \mathbf{R}_{\text{Listing}} \mathbf{v}_{\text{p.p.}} \ , \tag{4}$$

$$\boldsymbol{\omega} = \mathbf{R}_{\text{Listing}} \boldsymbol{\omega}_{\text{p.p.}} \ . \tag{5}$$

To be able to estimate the point-of-gaze, a set of subject-specific eye parameters has to be obtained through a personal calibration procedure. Suppose now that the point-of-gaze estimation method being used can estimate $\mathbf{c}$ and $\boldsymbol{\omega}$ without knowing the value of any subject-specific eye parameter [Shih and Liu 2004; Guestrin and Eizenman 2006; Guestrin and Eizenman 2008; Nagamatsu et al. 2008; Guestrin 2010]. In such case, only the subject-specific deviation of the visual axis from the optic axis of the eye needs to be determined. Finding this deviation is equivalent to finding the direction of the optic axis of the eye when the eye is in the primary position ($\boldsymbol{\omega}_{\text{p.p.}}$).

In the personal calibration procedure, $\mathbf{c}$ and $\boldsymbol{\omega}$ are determined while the subject fixates a known point $\mathbf{g}$ (not necessarily in the primary position). The unit vector in the direction of the visual axis is then given by

$$\mathbf{v} = \frac{\mathbf{g} - \mathbf{c}}{\left\| \mathbf{g} - \mathbf{c} \right\|} \ . \tag{6}$$

Substituting (6) into (2)-(3) provides the direction and magnitude of the rotation vector prescribed by Listing's law when the subject fixated the target calibration point. After obtaining the corresponding rotation matrix, $\mathbf{R}_{\text{Listing}}$, using Rodrigues' rotation formula, $\boldsymbol{\omega}_{\text{p.p.}}$ is obtained from (5) as

$$\boldsymbol{\omega}_{\text{p.p.}} = \mathbf{R}^{T}_{\text{Listing}} \boldsymbol{\omega} \ , \tag{7}$$

using the fact that rotation matrices satisfy $\mathbf{R}^{-1} = \mathbf{R}^{T}$.

For the estimation of the point-of-gaze, the next step after determining $\mathbf{c}$ and $\boldsymbol{\omega}$ consists of finding $\mathbf{v}$ given $\boldsymbol{\omega}$, $\boldsymbol{\omega}_{\text{p.p.}}$ and $\mathbf{v}_{\text{p.p.}}$. Following the reasoning from [Nagamatsu et al. 2008], the direction and magnitude of the rotation vector prescribed by Listing's law can be found from $\boldsymbol{\omega}$, $\boldsymbol{\omega}_{\text{p.p.}}$ and $\mathbf{v}_{\text{p.p.}}$ as

$$\boldsymbol{\lambda} = \frac{\mathbf{v}_{\text{p.p.}} \times (\boldsymbol{\omega} - \boldsymbol{\omega}_{\text{p.p.}})}{\left\| \mathbf{v}_{\text{p.p.}} \times (\boldsymbol{\omega} - \boldsymbol{\omega}_{\text{p.p.}}) \right\|} \ , \tag{8}$$

$$\varepsilon = \arccos\left[\frac{\boldsymbol{\omega}_{\text{p.p.}} \bullet \boldsymbol{\omega} - (\boldsymbol{\omega}_{\text{p.p.}} \bullet \boldsymbol{\lambda})^2}{1 - (\boldsymbol{\omega}_{\text{p.p.}} \bullet \boldsymbol{\lambda})^2} \right] . \tag{9}$$

As before, the corresponding rotation matrix, $\mathbf{R}_{\text{Listing}}$, can be readily obtained using Rodrigues' rotation formula. Having found $\mathbf{R}_{\text{Listing}}$, $\mathbf{v}$ is obtained with (4). Having $\mathbf{c}$ and $\mathbf{v}$, the point-of-gaze is found using (1).

4 Experimental Results

Experiments were performed with a system that uses two video cameras and multiple infrared light sources to estimate the point-of-gaze on a computer screen from the pupil and corneal reflections after each subject completes a single-point personal calibration procedure [Guestrin and Eizenman 2008; Guestrin 2010]. The plane of the screen was vertical and, in the primary position, the subject's face was parallel to the screen (consequently, $\mathbf{v}_{\text{p.p.}}$ was perpendicular to the screen).

The experiments were carried out with three adults without eyeglasses or contact lenses in binocular conditions (binocular conditions correspond to most gaze estimation applications). The head of each subject was placed at 65 cm (nominal distance), 60 cm (minimum distance) and 70 cm (maximum distance) from the screen. For each head position, each subject completed a number of trials (2 for Subjects 1 and 2, and 7 for Subject 3). In each trial, the subject was asked to sequentially fixate 25 target points (arranged in a 5-by-5 rectangular grid) on the computer screen. For each target point, 50 point-of-gaze estimates (2.5 seconds @ 20 estimates/second) were obtained for each eye (the point-of-gaze was estimated for both eyes simultaneously) using one of the methods described in [Guestrin 2010] (referred to as the *CNoPlInt-3CR + PB-EF* method). The subject-specific deviation of the visual axis from the optic axis was determined when the head was at the nominal distance from the screen with the plane of the face parallel to the screen and the subject fixated the target point at the center of the screen.

The RMS point-of-gaze estimation error obtained when the point-of-gaze was estimated ignoring the torsion of the eye and when it was estimated using Listing's and Donders' laws is provided in Table 1 for the three subjects. It can be observed that although in most cases the RMS error is smaller when Listing's and Donders' laws are used, the differences are in the order of 0.01º of visual angle, which is insignificant for any practical purpose (certainly much lower than the noise level of any existing remote gaze estimation system). Figure 1 shows the mean point-of-gaze estimates obtained in each trial for both eyes. Only minor differences can be observed for the point-of-gaze estimates in the bottom corners of the array of target fixation points.

Subject	Eye	Torsion ignored	Listing's & Donders' laws
1	L	4.82 mm (0.40º)	4.81 mm (0.40º)
	R	4.84 mm (0.40º)	4.80 mm (0.39º)
2	L	7.26 mm (0.58º)	7.29 mm (0.58º)
	R	6.62 mm (0.55º)	6.50 mm (0.54º)
3	L	6.79 mm (0.56º)	6.69 mm (0.55º)
	R	5.99 mm (0.49º)	5.94 mm (0.49º)

Table 1: Experimental RMS point-of-gaze estimation error.

5 Discussion and Conclusions

The experimental results presented in the previous section show that when the point-of-gaze was estimated using Listing's and Donders' laws there was no significant accuracy improvement relative to when eye torsion was ignored. This is largely due to the relatively small amount of eye torsion expected under our experimental conditions. For reference, note that, using Fick's system of axes [Carpenter 1977; Ferman et al. 1987a], when the head of the subject was at 65 cm from the screen and the subject fixated the bottom right target point, the longitude (pan) angle of the fixation line of the right eye was about 10-11º, the longitude angle of the fixation line of the left eye was about 15-16º, and the latitude (tilt) angle of the fixation line of both eyes was about –20º of visual angle. In such a case, the magnitude of the torsion of the eye about the fixation line predicted by Listing's law [Ferman et al. 1987a; Guestrin 2010] is about 1.9º and 2.7º, respectively. For a 5º deviation between the visual and optic axes, a 2.7º torsion would result in a difference of less than 3 mm between the point-of-gaze estimates obtained when eye torsion is calculated with Listing's and Donders' laws and when eye torsion is ignored. It is recognized, however, that for a larger range of eye rotation the torsion would be more significant and should be taken into account.

In general, the torsion of the eye depends on the position and orientation of the head (especially the orientation of the head with respect to gravity), the point-of-gaze and whether vision is monocular or binocular. The torsion of the eye is also affected by disorders of the visual and oculomotor systems.

Listing's and Donders' laws are, in the best case, only approximate. Listing's law does not apply when the head is not erect, when the head is non-stationary [Carpenter 1977] or in binocular conditions, especially when viewing a relatively close object (convergence of the fixation lines, vergence eye movements) [Carpenter 1977; Alpern 1969b]. In addition, Listing's and Donders' laws do not hold during smooth pursuit (eye movements that follow a moving target) [Ferman et al. 1987b].

Even under monocular viewing conditions when the head is erect and stationary and the point-of-gaze is also stationary, Listing's law is only approximate: physiological eye movements show considerable stochastic as well as systematical deviations from this law [Ferman et al. 1987a]. Some of the observations from [Ferman et al. 1987a] are: (a) even when the eye is in the primary position there can be considerable fluctuations of torsion; (b) there is asymmetry between the torsion in the nasal quadrants and the torsion in the temporal quadrants; (c) there are variations in torsion between the left and right eyes of any subject, and among subjects.

From the preceding discussion it is clear that Listing's and Donders' laws have several limitations for practical use. For example, while certain applications in ophthalmology and neurology require the estimation of gaze in monocular conditions (the eye that is not being tested is covered or patched) with a stationary head, most gaze estimation applications need to be carried out in binocular conditions under natural translational and rotational head movements. Moreover, the torsion of the eye resulting from lateral head tilt (observed often in applications with infants) can be significantly larger than the torsion predicted by Listing's law, and even have opposite direction. All this suggests that, to properly account for eye torsion, the torsion of the eye should be measured independently, for example, by tracking the iris pattern and/or the scleral blood vessels.

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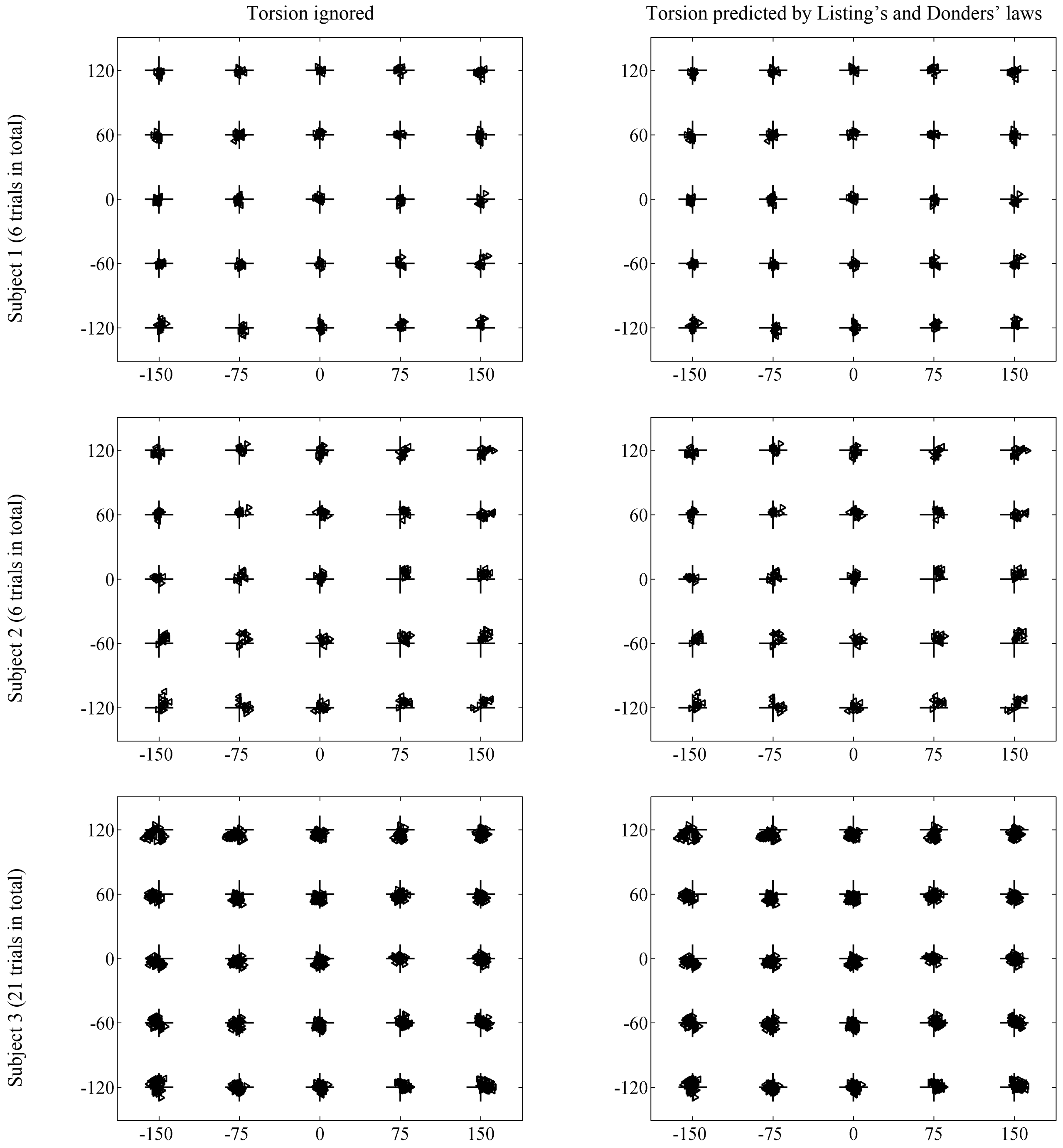

Figure 1: Mean point-of-gaze estimates (+ : target fixation point, ◁ : left eye, ▷ : right eye). All coordinates are in mm.